

# The Old VIX vs. New VIX.

Mark Ioffe

In 1993, the Chicago Board Options Exchange® (CBOE®) introduced the CBOE Volatility Index® .VIX® and it quickly became the benchmark for stock market volatility. The index calculations were based on paper by Professor Robert E. Whaley of Duke University. Accordingly this paper VIX® is constructed from implied volatilities of the eight near –the-money, nearby, and second nearby OEX option series. For calculation used formulas:

$$VIX = \sigma_1 \left( \frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}} \right) + \sigma_2 \left( \frac{22 - N_{t_2}}{N_{t_2} - N_{t_1}} \right)$$

$$\sigma_1 = \sigma_1^{X_l} \left( \frac{X_u - S}{X_u - X_l} \right) + \sigma_1^{X_u} \left( \frac{S - X_l}{X_u - X_l} \right)$$

$$\sigma_2 = \sigma_2^{X_l} \left( \frac{X_u - S}{X_u - X_l} \right) + \sigma_2^{X_u} \left( \frac{S - X_l}{X_u - X_l} \right)$$

$$\sigma_1^{X_l} = \frac{\sigma_{1,C}^{X_l} + \sigma_{1,P}^{X_l}}{2}$$

$$\sigma_2^{X_l} = \frac{\sigma_{2,C}^{X_l} + \sigma_{2,P}^{X_l}}{2}$$

$$\sigma_1^{X_u} = \frac{\sigma_{1,C}^{X_u} + \sigma_{1,P}^{X_u}}{2}$$

$$\sigma_2^{X_u} = \frac{\sigma_{2,C}^{X_u} + \sigma_{2,P}^{X_u}}{2}$$

(1)

Where

$X_l$  is exercise price just below the current index level S,  $X_u$  is exercise price just above the current index level S,  $\sigma_{1,C}^{X_l}$  is implied volatility of Call option with strike and time to expiration  $N_{t_1}$ ,  $\sigma_{1,P}^{X_l}$  is implied volatility of Put option with strike and time to expiration  $N_{t_1}$  and so on.

The fundamental features of new VIX introduced in 2003 remain the same. VIX continues to provide a minute-by-minute snapshot of expected stock market volatility over the next 30 calendar days. This volatility is still calculated in real time from stock index option prices and is continuously disseminated throughout trading day.

What is new? There are two important changes in the new VIX methodology. This is quote from description. "The most significant change is a new method of calculation. The new VIX estimates expected volatility from the prices of stock index options in a wide range of strike prices, not just at-the money strikes as in the original VIX. Also, the new VIX is not calculated from Black Sholes option pricing model; the calculation is independent of any model. The new VIX uses a newly developed formula to derive expected volatility by averaging the weighted prices of out- of- the money puts and calls. This simple and powerful derivation is based on theoretical results that have spurred the growth a new market where risk managers and hedge funds can trade volatility. And market makers can hedge volatility trades with listed options"

Unfortunately, cited above statements are not correct. As it follows from formulas (1) the original VIX estimates expected volatility not just at-the money strikes, but from implied volatilities of the eight near –the-money, nearby, and second nearby OEX option series.

Also, we will show that the new VIX implicitly is strongly depends on Black Sholes option pricing model. Namely, the generalized formula used in the new VIX calculation calculates volatility of rate of index return if and only if future index value has log-normal distribution (Black Sholes model). We consider portfolio comprising infinite large amount of Call and Put options. For strikes lesser than some

quantity  $F$  we include in portfolio  $\frac{dK}{K^2}$  options Put for any strike  $K$ . For strikes more than some quantity  $F$  we include in portfolio the same number options Call for any strike  $K$ . This portfolio price  $Port$  equals:

$$Port = \int_0^F \frac{1}{k^2} P(k) dk + \int_F^\infty \frac{1}{k^2} C(k) dk$$

**(2)**

Where  $P(k)$ ,  $C(k)$  – premiums Put и Call, calculated by formulas:

$$P(K) = e^{-rT} \int_0^K (K - S) p(S) dS$$

$$C(K) = e^{-rT} \int_K^\infty (S - K) p(S) dS$$

**(3)**

In formulas (3)

$r$  = risk-free interest rate to expiration

$T$  = time to expiration

$P(S)$  = probability density of index

By substituting (3) в (2), we have

$$Port = e^{-rT} (Port1 + Port2)$$

$$Port1 = \int_0^F \frac{1}{k^2} dk \int_0^k (k - S) p(S) dS$$

$$Port2 = \int_F^\infty \frac{1}{k^2} dk \int_k^\infty (S - k) p(S) dS$$

**(4)**

By changing order of integration in (7) we get

$$Port1 = \int_0^F p(S) dS \int_S^F \frac{k - S}{k^2} dk$$

$$Port2 = \int_F^\infty p(S) dS \int_F^S \frac{S - k}{k^2} dk$$

**(5)**

It follows from (5)

$$Port = e^{-rT} \int_0^\infty p(S) dS \int_S^F \frac{k - S}{k^2} dk = e^{-rT} \int_0^\infty p(S) (\ln(F) - 1 + \frac{S}{F} - \ln(S)) dS$$

**(6)**

Because  $p(S)$  = probability density formula (6) can be written as

$$Port = e^{-rT} (\ln(F) - 1 + \frac{E(S)}{F} - \int_0^\infty p(S) \ln(S) dS)$$

**(7)**

Where

$E(S)$ =math. expectation S at expiration time

Until now we did no assumptions about statistical distribution index price. Now we suppose index price is simulated by continuous diffusion process

$$\frac{dS(t)}{S(t)} = (r - q)dt + \sigma dW(t)$$

**(8)**

where

$W(t)$ =Wiener process

$r$ =interest rate

$q$ =dividend yield

$\sigma$ =volatility

It follows from (8) that variable S is log-normal variant and its math. expectation equals

$$E(S) = S_0 e^{(r-q)T}$$

**(9)**

Also, it follows from (8) that integral in (7) is math. expectation of corresponding normal variable, i.e.

$$\int_0^{\infty} p(S) \ln(S) dS = \ln(S_0) + (r - q)T - 0.5\sigma^2 T$$

**(10)**

where

$S_0$ =today index price

By substituting (9) и (10) into (7) we get

$$Port = e^{-rT} \left( \ln\left(\frac{F}{S_0}\right) - 1 + \frac{S_0 e^{(r-q)T}}{F} - (r - q)T + 0.5\sigma^2 T \right)$$

**(11)**

Using (11) we can calculate variance through portfolio value:

$$\sigma^2 = \frac{2}{T} \left( e^{rT} Port + 1 + \ln\left(\frac{S_0}{F}\right) - \frac{S_0 e^{(r-q)T}}{F} + (r - q)T \right)$$

**(12)**

By substituting (2) into (12) we get

$$\sigma^2 = \frac{2}{T} e^{rT} \int_{-\infty}^{\infty} \frac{CP(K)}{K^2} dK + \frac{2}{T} \left( 1 + \ln\left(\frac{S_0}{F}\right) - \frac{S_0 e^{(r-q)T}}{F} + (r - q)T \right)$$

$$CP(K) = \begin{cases} C(K) \dots \text{if} \dots K > F \\ P(K) \dots \text{if} \dots K < F \end{cases}$$

$C(K)$  = Call... Price

$P(K)$  = Put... Price

**(13)**

Let F will be theoretical future index value, i.e.

$$F = S_0 e^{(r-q)T}$$

**(14)**

By substituting (14) into (13) we get

$$\sigma^2 = \frac{2}{T} e^{rT} \int_{-\infty}^{\infty} \frac{CP(K)}{K^2} dK$$

$$CP(K) = \begin{cases} C(K) \dots \text{if } K > F \\ P(K) \dots \text{if } K < F \end{cases}$$

$C(K)$  = Call..Price

$P(K)$  = Put...Price

**(15)**

Formula (13) is true if and only if future index has log-normal distribution. But on this formula is based the generalized formula used in the new VIX calculation:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i} e^{rT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2$$

**(14)**

Where

F=forward index level derived from index option prices

$K_i$ =Strike price of  $i$ th out-of-the money option; a Call if  $K_i > F$  and Put if  $K_i < F$

$\Delta K_i$ =interval between strike prices-half the distance between the strike on either side of

$K_0$ =first strike below the forward index level, F

$Q(K_i)$ =the midpoint of the bid-ask spread for each option with strike .

It is clear that first term in formula (14) is numerical calculation by method of rectangles of integral in formula (13). Second term must comply with second term in formula (13) for certain selection of parameters F and

. For at-the money option  $K_0 = S_0$ . If F will be theoretical future index value, see (14) then second term equals:

$$\frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2 = \frac{1}{T} \left( e^{(r-q)T} - 1 \right)^2 \approx (r-q)^2 T \ll 1$$

**(15)**



307 East 53rd Street, 6th Floor  
New York, NY 10022

By Phone: 212-223-3552  
By Fax: 212-421-6608

By Email: [info@egartech.com](mailto:info@egartech.com)  
[www.egartech.com](http://www.egartech.com)



307 East 53rd Street, 6th Floor  
New York, NY 10022

By Phone: 212-223-3552  
By Fax: 212-421-6608

By Email: [info@egartech.com](mailto:info@egartech.com)  
[www.egartech.com](http://www.egartech.com)



307 East 53rd Street, 6th Floor  
New York, NY 10022

By Phone: 212-223-3552  
By Fax: 212-421-6608

By Email: [info@egartech.com](mailto:info@egartech.com)  
[www.egartech.com](http://www.egartech.com)