CLIQUE OPTION PRICING

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Abstract

We show how can be calculated Clique option premium. If number of averaging dates enough great we use central limit theorem for stochastic variables and derived analytical formula for option price. For small number of averaging we use 2 methods: Monte Carlo and method based on Gauss-Legendre formula for numerical integration.

We show how can be calculated Clique option premium. If number of averaging dates enough great we use central limit theorem for stochastic variables and derived analytical formula for option price. For small number of averaging we use 2 methods: Monte Carlo and method based on Gauss-Legendre formula for numerical integration.

We consider option with pay-off P determined by formula:

$$P = MAX\{0\%, \sum_{i=1}^{i=n} MIN[x\max\%, MAX(\frac{SPX_i}{SPX_{i-1}} - 100\%, x\min\%)] - Offset\%\}$$

(1)

where
SPX_k-value of index S&P500 at time k month from today (k=12 or 36 for example)
xmax %-maximum limit in percent, (xmax=1.5% for example)
xmin %-minimum limit in percent, (xmin=-1.0 % for example)

This option we will name Clique option.

The rate of return SPX r_k for time from t_{k-1} through t_k is calculated by formula:

$$r_k = Log(1 + \frac{SPX_k}{SPX_{k-1}} - 1) \cong \frac{SPX_k}{SPX_{k-1}} - 1$$

(2)

From (1) and (2) we have

$$P = MAX\{0\%, \sum_{k=1}^{k=n} MIN[x\max\%, MAX(r_k\%, x\min\%)]\} - Offset\%$$

(3)

We shall assume that the monthly logarithmic returns r_k (k=1,2…n) are values at times t_k (k=1,2…n) of continuous diffusion process:

$$dr(t) = \mu(t)dt + \sigma(t)dW(t)$$

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(4)

where W(t)-standard Wiener process

It follows from (4) that r_k (k=1,2…n) are independent stochastic variables and each r_k follows Gaussian distribution, that is, the probability density function of r_k is given by

$$f(x) = \frac{1}{\sigma_k \sqrt{2\pi}} e^{\frac{1}{2\sigma_k^2}(x-\mu_k)^2} \equiv N(x, \mu_k, \sigma_k)$$

$$\mu_k = \int_{t_{k-1}}^{t_k} \mu(t)dt$$

$$\sigma_k = \sqrt{\int_{t_{k-1}}^{t_k} \sigma^2(t)dt}$$

(5)

This model is fully determined by two deterministic functions, namely the average μ(t) and the variance σ(t).

Let stochastic variable xk is determined by formula:

$$x_k = MIN[x\max\%, MAX(r_k\%, x\min\%)]$$

(6)

From (3) and (5) it follows that cluque option payoff is:

$$P = \max(0, \sum_{k=1}^{k=n} x_k - offset)$$

(7)

From aforementioned assumption we find that xk (k=1,2…n) are independent stochastic variables , and are distributed with probability density function g(x) given by:

$$g(x) = p\min \Delta(x - x\min) + p\max \Delta(x - x\max) + I(x\min, x\max) N(x, \mu_1, \sigma_1)$$

$$u1 = \frac{x\min - \mu_1}{\sigma_1}$$

$$u2 = \frac{x\max - \mu_1}{\sigma_1}$$

$$p\min = Lp(u1)$$

$$p\max = 1 - Lp(u2)$$

$$Lp(u) = \int_{-\infty}^{u} N(u,0,1)du - Laplac's..function$$

$$\Delta() - Dirac's...function$$

$$I(x) = 1..if...x \in [x\min, x\max]$$

$$I(x) = 0...if...x \notin [x\min, x\max]$$

(8)

First moment Ex (average value), second moment Ex^2 and variance Dx for any g(x) distribution can be calculated by formulas:

$$u1 = \frac{x\min - \mu}{\sigma}$$
$$u2 = \frac{x\max - \mu}{\sigma}$$
$$p1 = Lp(u1)$$
$$p2 = 1 - Lp(u2)$$
$$Eu = p1u1 + p2u2 + N(u1,0,1) - N(u2,0,1)$$
$$Eu^2 = p1u1^2 + p2u2^2 + u1\,N(u1,0,1) - u2\,N(u2,0,1) + Lp(u2) - Lp(u1)$$
$$Du = Eu^2 - (Eu)^2$$
$$Ex = \sigma Eu + \mu$$
$$Dx = \sigma^2 Du$$

(9)

Non-arbitrage price clique option Pr is mathematical expectation (average) of payoff function. From (7) we have:

$$y = \sum_{k=1}^{n} x_k$$
$$\Pr = E(MAX(0, y - ofset))$$

(10)

When n >>1 (enough great) on base central limit theorem for stochastic variable y good approximation is Gaussian distribution with parameters the average μS and the variance σS. Obviously,

$$\mu S = \sum_{k=1}^{k=1} \mu_k$$
$$\sigma S = \sqrt{\sum_{k=1}^{k=n} \sigma_k^2}$$

(11)

For this approximation non-arbitrage price clique option PrD with discounting factor rd can be calculated by formula:

$$\Pr D = e^{-rd(n)}(\sigma S^2 N(0, \mu S - offset, \sigma S) + (\mu S - offset)\,Lp(-\frac{\mu S - offset}{\sigma S}))$$

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The one practical way to calculate price clique option for not great n is Monte Carlo (MC) method. But what n we can assume as great? This question can have only empirical decision. We can calculate option's price by two methods and compare results. For MC method we calculated not only price PrMC value but low PrMCLow and upper PrMCUP values bounds. When value, calculated by approximation using Gauss distribution, hits MC interval between low and upper values we suppose that n is great and we can use approximation. The example of this calculation is shown in Table 1.

Table 1

n	Pr	PrMCLow	PrMC	PrMCUP
1	0.765297717	1.098355509	1.1594291	1.220503
2	1.493950077	1.168343006	1.2633389	1.358335
3	2.120094054	2.025776208	2.1414485	2.257121
4	2.696498232	2.676506968	2.8086672	2.940827
5	3.246241653	2.851414989	3.0083096	3.165204
6	3.776842044	3.475001792	3.6461891	3.817376

In Table 1 values relates to n month from today clique options with parameters: xmin=-1.5%, xmax=3%, offset=2%, rd=2%, means and volatility are constant and mu=0 and vol=30%. From Table 1 we can see that for n=6 Pr arranges between low and upper bounds, i.e. for this case n=6 is "great" number. Excel function CliqueNewGen(xmin,xmax , offset, t, mu0, vol, rd, tday, MaxMC) calculates clique option price by shown above method. The input parameter MaxMC is maximum integer for using Monte Carlo method. Default value MaxMC is 6. The good approximation for mu0 (default value) is zero.
The other method of clique option price calculation for small n is following. Fourier transform of the probability density function (characteristic function) of stochastic variable xk φ(t) can be calculated by formula:

$$\varphi(t) = \int_{-\infty}^{\infty} e^{itx} g(x)dx = p\min e^{itx\min} + p\max e^{itx\max} + \int_{x\min}^{x\max} e^{itx} N(x, \mu_k, \sigma_k)dx$$

(13)

Using for calculation integral in (13) Gauss-Legendre formula provides:

$$\varphi(t) = \int_{-\infty}^{\infty} e^{itx} g(x)dx \cong p\min e^{itx\min} + p\max e^{itx\max} + \sum_{i=1}^{N} W(i) e^{itx(i)} N(x(i), \mu_k, \sigma_k)$$

(14)

where:
x(i)-abscissas
W(i)-weights.

From formula (14) it follows stochastic variable x_k can be seen as discrete stochastic variable with values $x_k(1)$=xmin, $x_k(2)$=x(1), $x_k(3)$=x(2)……. $x_k(N+1)$=x(N), $x_k(N+2)$=xmax and discrete probabilities are:

$$p_k(1) = p\min$$
$$p_k(i) = W(i-1)N(x(i-1), \mu_k, \sigma_k)$$
$$p_k(N+2) = p\max$$

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Using this representation of stochastic variable x_k as discrete stochastic variable, option price PrS calculation, for example, for n=3 provided by formula:

$$C = \sum_{i=1}^{N+2}\sum_{j=1}^{N+2}\sum_{l=1}^{N+2} \max(x(i) + x(j) + x(l) - offset, 0) p(i) p(j) p(l)$$

$$\Pr S = Ce^{-r\Delta t}$$

(16)

Results of calculation by this method are shown in Table 2.

Table 2

N	PrS
1	1.12615576
2	1.280115756
3	2.13936519
4	2.802662383
5	3.160525718
6	3.76590872

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