BOOST OPTIONS PRICING

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Abstract

We describe method for pricing of BOOST option. Method based on solving of partial differential equation (PDE) by using Fourier transformation. We reduce pricing problem to numerical integration. For numerical integration we use Simpson's method.

In occupation time derivitvie with double barriers, the derivative payoff depend of the amount of occupation time that the stock price stays within certain range or corridor. Societe Generale introduced the option termed BOOST (Banking On Overall Stability). The option is characterized by corridor $[S_{\min}, S_{\max}]$ for stock price process. At maturity the holder receives a payoff that is proportional to the total occupational tine that the stock price stays within the corridor since inception. We considere how one can calculate this option price.

Let $x = \frac{1}{\sigma} LOG(\frac{S}{S_0})$

where x is a variable in a "Black-Scholes" environment, σ is volatility, S0 is initial stock value.
That is

$$dx = \gamma dt + dw(t)$$

Where
γ=drift;
w(t)=Wiener process.
Value γ equals

$$\gamma = \frac{r_d - r_f - 0.5\sigma^2}{\sigma}$$

Where
rd=dividend yield
rf=riskless rate
Let $\tau(x,t)$ is time interval when inequalities

$$x_{\min} \le x(t) \le x_{\max}, 0 \le t \le T,$$

are true (for t=0 x(0)=x).
xmin , xmax equals

$$x_{\min} = \frac{1}{\sigma} LOG\left(\frac{S_{Low}}{S_0}\right)$$

$$x_{\max} = \frac{1}{\sigma} LOG\left(\frac{S_{Up}}{S_0}\right)$$

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Where

S_{Low} =lower barrier;

S_{Up} =upper barrier;

Function $\tau(x,t)$ is full time ,when x(t) was inside two barriers $x_{\min} and x_{\max}$ .It may go out and come back in that barriers.

Let $f(x,t) = M(\tau(x,t))$ is an average time, math. expectation,,when value x(t) was inside two barriers $x_{\min} and x_{\max}$.

One can see that function f(x,t) obeyes this non-homogeneous linear partial differencial equation (PDE):

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial f}{\partial x} + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} + \chi(x_{\min}, x_{\max})$$

where

$$\chi(x_{\min}, x_{\max}) = \left\{ \frac{1, if x_{\min} \le x \le x_{\max}}{0} \right.$$

(1)

For equation (1) we will to solve the Cauche problem with known initial value:

f(x,0)=0

(2)

Because,obviously, f(x,t)→0 for x→ ±∞, we will to solve problem by using Fourier transformation
But at first we do substitution of independent variables by formulas:

$$x = \frac{x_{\max} - x_{\min}}{2} y + \frac{x_{\max} + x_{\min}}{2}$$

$$\tau = \frac{4t}{(x_{\max} - x_{\min})^2}$$

(3)

With new variables equation (1) will be

$$\frac{\partial f}{\partial \tau} = \gamma 1 \frac{\partial f}{\partial y} + \frac{1}{2}\frac{\partial^2 f}{\partial y^2} + l^2 \chi(-1,1)$$

(4)

$$l = \frac{x_{\max} - x_{\min}}{2}$$

$$\gamma 1 = \frac{x_{\max} - x_{\min}}{2} \gamma$$

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(5)

Let

$$\bar{f}(\tau,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(\tau,y)e^{i\varpi y}dy$$

(6)

From identity Fourier

$$f(\tau,y)=\int_{-\infty}^{\infty}\bar{f}(\tau,\varpi)e^{-i\varpi y}d\varpi$$

(7)

Substitution (6) in (4) gives

$$\frac{\partial\bar{f}}{\partial\tau}=-\left(i\gamma 1\varpi+\frac{1}{2}\varpi^2\right)\bar{f}+\frac{l^2}{\pi}\frac{\sin\varpi}{\varpi}$$

(8)

Because $\bar{f}(0,\varpi)=0$ from (8) we have

$$\bar{f}(\tau,\omega)=\frac{l^2\sin\omega(\frac{1}{2}\omega-i\gamma 1)}{\pi\omega^2\left(\gamma 1^2+\frac{1}{4}\omega^2\right)}\left(1-e^{-\frac{1}{2}\omega^2\tau}(\cos\omega\tau\gamma 1-i\sin\omega\tau\gamma 1)\right)$$

(9)

From (9) we found that real and imaginary parts function $\bar{f}(\tau,\varpi)$ are

$$\operatorname{Re}\bar{f}=\frac{l^2\sin\omega}{\pi\omega^2\left(\gamma 1^2+\frac{1}{4}\omega^2\right)}\left(\frac{1}{2}\omega-e^{\frac{1}{2}\omega^2\tau}\left(\frac{1}{2}\omega\cos\omega\tau\gamma 1-\gamma 1\sin\omega\tau\gamma 1\right)\right)$$

(10)

$$\operatorname{Im}\bar{f}=\frac{l^2\sin\omega}{\pi\omega^2\left(\gamma 1^2+\frac{1}{4}\omega^2\right)}\left(-\gamma 1+e^{\frac{1}{2}\omega^2\tau}\left(\gamma 1\cos\omega\tau\gamma 1+\frac{1}{2}\omega\sin\omega\tau\gamma 1\right)\right)$$

(11)

Let

$$\sin\varphi=\frac{\omega}{\sqrt{4\gamma 1^2+\omega^2}}$$

$$\cos\varphi=-\frac{\gamma 1}{\sqrt{4\gamma 1^2+\omega^2}}$$

(12)

With (12) equation (10) and (11) will be

$$\operatorname{Re}\bar{f}=\frac{l^2\sin\omega}{\pi\omega^2\left(\gamma 1^2+\frac{1}{4}\omega^2\right)}\left(\sin\phi-e^{-\frac{1}{2}\omega^2\tau}\sin(\phi+\omega\tau\gamma 1)\right)$$

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(13)

$$IM\bar{f} = \frac{l^2 \sin\omega}{\pi\omega^2\left(\gamma 1^2 + \frac{1}{4}\omega^2\right)}\left(\cos\phi - e^{-\frac{1}{\,}\omega^2\tau}\cos(\phi + \omega\tau\gamma 1)\right)$$

(14)

From formuls (7),(13),(14) we found that

$$f(\tau, y) = \frac{2l^2}{\pi}\int_0^{\infty}\frac{\sin\omega}{\omega^2\sqrt{\gamma 1^2 + \frac{1}{4}\omega^2}}\left(\sin(\varphi + \omega y) - e^{-\frac{1}{2}\omega^2\tau}\sin(\varphi + \omega y + \omega\gamma 1\tau)\right)d\omega$$

(15)

We need value of integral for values

$$\tau = \frac{4T}{(x_{\max} - x_{\min})^2}$$

(16)

and

$$y = \frac{x_{\max} + x_{\min}}{x_{\min} - x_{\max}}$$

(17)

For calculation integral (15) we used numerical integration by Simpson formula. We got interval of integration [0,20].
Example.
We use the following numbers:
S0 = 1.52
T = 92 days
rd = 5,85%
rf = 5,90%
Vol = 12.5%

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The Table1 shows values of Boost options for different levels of low boundary (H) and upper boundary(F)
Table1

H	F	F- H	Boost
1.5175	1.5225	0.005	0.010215055
1.515	1.525	0.01	0.020096161
1.5075	1.5325	0.025	0.047807755
1.495	1.545	0.05	0.087981981
1.47	1.57	0.1	0.148894661
1.42	1.62	0.2	0.214801998
1.37	1.67	0.3	0.238927271
1.32	1.72	0.4	0.246113022
1.27	1.77	0.5	0.247885615
1.22	1.82	0.6	0.248309943
0.77	2.27	1.5	0.248203187

The table 2 shows values of Boost options for different levels of Volatility and Time to expiration
H = 1.42, F=1.62

Table 2

Time Vola	30	92	180
	Boost	Boost	Boost
5%	0.08167506	0.247812	0.470902
12.5%	0.08025946	0.214802	0.356229
20%	0.07297981	0.171348	0.266072
30%	0.06148337	0.131242	0.195822

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