About article “Unique Option Pricing Measure With Neither Dynamic Hedging nor Complete Markets” by Nassim Nicholas Taleb †_School of Engineering, NYU, & Former Option Trader

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The article presents the author’s attempt to find a way to estimate the price of option other than the well-known and common Black-Scholes method, which reduces to the Black-Scholes formula for European Call option and the numerical solution of the corresponding parabolic differential equations for the American option. The author believes that for derivation of the Black-Scholes formula should be used not only dynamic hedging, but also some other economic assumptions. In fact, the Black Scholes formula can be easily obtained from the assumption that the process of change in stock price is described by the well-known geometric random walk process with constant values of coefficient of trend and diffusion and without dynamic hedging and other economic assumptions. (See [1])

The author writes, ’that the heuristics used by traders for centuries are both more robust, more consistent, and more rigorous than held in the economics literature.’ And ’we have replaced the complexity and intractability of dynamic hedging with a simple, more benign interpolation problem, and explained the performance of pre-Black-Scholes option operators using simple heuristics and rules, bypassing the structure of the theorems of financial economics.’ It is interesting what kind of heuristics author is talking about? When using the Black-Scholes estimation method of option pricing is reduced to the evaluation of the so-called volatility based on market data. Unfortunately, the author, who is former option trader, in his article did not disclose any formalism used by traders and what is benign interpolation problem used for option pricing. In particular, it is unclear what kind of distribution with finite first moment and infinite second he is talking about.

In conclusion, we present some purely technical or rather mathematical comments that however significantly affect the understanding and appreciation of the essence of the article.

Puzzling formula (3) and (4), in which it is assumed that the Put option has a one probability distribution, measure, and Call get other distribution. And even to this assumption of formula (3) and (4) are written incorrectly, because in Lebesgue integral a function used the same argument – the stock price. Most likely, the formula for option pricing C and P should be written in the form:

\[ C = \int_0^\infty c d\mu_c(c) \]
\[ P = \int_0^\infty p d\mu_p(p) \]

Where

\( \mu_c(c) \) – measure for Call;
\( \mu_p(p) \) – measure for Put.
It should be noted that the Put-Call Parity, in which forward and strike participate are easily obtained from the assumption that the option prices are averages of deviation of stock prices from the strike, provided that these deviations are positive. If we assume that the distributions of shares for Put-Call are different, it is unclear how generally to get Put-Call Parity.

Obviously,

\[
\frac{C(S_{t0}, K, t) - C(S_{t0}, K + \Delta K, t)}{\Delta K} = \frac{1}{\Delta K} \int_{K}^{K+\Delta K} (S - K) \mu(S) + \int_{K+\Delta K}^{\infty} \mu(S)
\]

From this equation it follows that the equation (6) given in the article is not always correct, but only at certain smoothness measures, i.e., in particular, in the absence of jumps. Clearly, if the measure is not continuous, equations (7) and (8) will not be correct.

References.