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Phone: +1 212 223 3552 • Fax: +1 212 421 6608
info@egartech.com • www.egartech.com

Pricing of Asian option with Matlab

Mark Ioffe

Abstract

The article refers to the calculation of the price of Asian option in Matlab. For reasons not completely understood, Matlab uses the so-called binomial option pricing model estimation. We show that for the calculation of the price of Asian option in Matlab we have to spend much less time using the method of Levi-Turnbull [1] than in case of using binomial method with no less accuracy.

To assess the price of Asian option a lot of developed methods and tools are available now. Apparently, the most common and reliable method is based on the approximation of the distribution of Arithmetic Mean of log-normal values as the log-normal distribution proposed by Levy and Turnbull [1]. This method is analytical, i.e. it is not required for its implementation the use of any kind of finite-difference schemes that in financial mathematics are considered as the binomial, trinomial etc. model. We tried to analyze how Asian option estimated in a very popular and high-quality package Matlab, in its section Financial Derivatives Toolbox.

For reasons not completely understood, Matlab is using the so-called binomial option pricing model estimation. Mathematical essence of this method is described in detail in [2], where it is shown that this method is simple and obvious finite-difference method used in numerical analysis for solving differential equations with partial derivatives. Moreover, this method has a limited stability, which is particularly evident in the case of options with barriers [3]. Also, this method is referred to as the continuous arithmetic averaging, that is the average value is determined by the formula:

$$A = \frac{1}{T}\int_0^T S(\tau)d\tau \quad (1)$$

Where
S (t) – stock price.

Thus, in determining of option such an important from the practical point of view parameter as frequency averaging (day, week, month, etc.) is missing. From this point of view, Levi-Turnbull method is undoubtedly preferable, because the frequency averaging is one of the inputs into calculation parameters.

In general, the choice of a method for estimating the price of the option is determined by two criteria: accuracy of the method and the time required for the calculation. Error of the method is determined by the difference between "true" and the estimated value of the option. Determination of the “true” value of an option is possible only for a very limited set of options. In many cases, including

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consideration of Asian option the only way to determine the "true" value is a Monte Carlo method. It is known that this method for obtaining an acceptable result requires significant computer time. But it usually has no alternatives.

The Monte Carlo method for calculating the price of Asian option was implemented in Matlab as follows. In accordance with the Black-Sholes model, the stock price S (t) is a log-normal random process That is:

$$S(t) = e^{x(t)}$$

(2)

The random variables x(t) form a normal, Gaussian stochastic process with the vector of the expectation of m(t) and covariance matrix Covmat (t1, t2), calculated according to formulas:

$$m(t) = LnS_0 + (r_d - r_f - 0.5\sigma^2)t = LnS_0 + \gamma t$$
$$Covmat(t1,t2) = \sigma^2 Min(t1,t2)$$

(3)

Where
S_0 – initial stock price value;
r_d – constant interest rate;
r_f – constant dividend rate;
σ – constant volatility.

Calculating the price of the option by the Monte Carlo method, implemented in the function AsianMC (see. Appendix) consists of the following steps. First, using the Matlab procedure Singular value decomposition, we present the covariance matrix Covmat in the form:

$$Covmat = U * S * V$$
$$U * V = E$$

(4)

Where
U, V – unitary matrices;
S – diagonal matrix;
E – unit matrix.

Diagonal elements of S equals:

$$diagS(i) = \sigma^2 t(i)$$
$$i = 1,2...n$$

(5)

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Where
n – number of averaging points.

After this, using the Matlab function normrnd (mu,sigma) we calculate a vector of random numbers R chosen from n-dimensional normal distribution with parameters m(t) , Covmat (t1,t2):

$$R = U * \text{normrnd}(\text{zeros}(n1,1), \text{sqrt}(\text{diag}(S))) + m(t(i));$$

(6)

Components of R are the values of the logarithms of stock prices needed to calculate the Arithmetic Mean, in turn, required for option pricing. Thus, we have for Call option:

$$C = Max(\frac{1}{n}\sum_{i=1}^{n} e^{R(i)} - K, 0)$$

(7)

Where
K – Strike.

By performing the above definition of the random variable C sufficiently large number of times M, we can find the option price as the average value of the sample, as well as to calculate the standard deviation of the sample. Under the condition of normality, we can estimate the confidence interval, i.e. the maximum and minimum values obtained for the price. Thus,

$$\text{Price} = \frac{1}{n}\sum_{i=1}^{M} C(i)$$
$$\sigma = \frac{1}{n-1}\sum_{i=1}^{M} (C(i) - \text{Price})^2$$
$$\text{PriceMax} = \text{Price} + 3\sigma$$
$$\text{PriceMin} = \text{Price} - 3\sigma$$

(8)

In formulas (8) probability that Price is within PriceMin and PrriceMax equals 0.995.

Calculating the price of the option by Levi-Turnbull method, implemented in the function Levi (see. Appendix).

Matlab calculates Asian option price using function `asianbycrr` (see Appendix).

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Numerical Results

Calculations were done for Asian option with following parameters. The stock price is 100, the strike price is 100, the risk-free rate is 10%, the dividend rate is 0%, the time to maturity is 1 year, the volatility of the underlying stock is 30% and frequency averaging 1 week (7 days).
The calculation results are shown in Table 1.

Table 1

	Levi-Turnbull	Matlab	Mcmax	MC	Mcmin
Price	8.8458	9.1866	8.9057	8.7879	8.6701
Time	0.658 s	8.361 s			

In Table 1, along with the prices the time of calculation is given in seconds for two variants of calculations.

Conclusion

As can be seen from Table 1, to calculate the price of Asian option in Matlab we have to spend much less time using method of Levi-Turnbull than in case of using binomial method with no less accuracy. Therefore, from a practical point of view for Asian option pricing in Matlab Levi-Turnbull method should be used.

Reference

[1] Levi, Edmond and Turnbill, Stuart “Average Intelligence”, Risk, February 1992.
[2] Mark Ioffe “Binomial Option Pricing Model”, EGAR Technology. 2013
[3] G. Ioffe, M.Ioffe “APPLICATION OF FINITE DIFFERENCE METHOD FOR PRICING BARRIER OPTIONS”, EGAR Technology. 2003

Appendix

```
function Levi

close all;

clear;

c0=clock;

Sigma = 0.30;

AssetPrice = 100;

DividendType = 'cash';
```

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```
DividendAmounts = [0; 0; 0; 0;];
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';
'01-Oct-2003'};

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates);

RateSpec = intenvset('Rates', 0.09, 'StartDates',...
'01-Jan-2003', 'EndDates', '31-Dec-2003');
ValuationDate = '1-Jan-2003';
Maturity = '01-Jan-2004';
TimeSpec = crrtimespec(ValuationDate, Maturity, 100);
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec);
%treeviewer(CRRTree);
OptSpec = 'call';
Strike = 100;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
Price = asianbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)
c1=clock;
[ca,pt]=Levi(100,100,0.09,0,0.3,'01-Jan-2003','01-Jan-2003','01-Jan-2004',7)
c2=clock;
t1=(c1(6)-c0(6))/1
t2=(c2(6)-c1(6))/1

function [ca,pt]=Levi(s0,k,rd,rf,sigma,t0,tstart,texp,delt)
topt = (datenum(texp) -datenum( t0)) / 365;
nav = floor((datenum(texp) - datenum(tstart)) / delt) + 1;
for i = 1:nav
```

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```
tt = (datenum(tstart) -datenum(t0) + (i - 1) * delt) / 365;
tt = max(tt, 1 / 365);
t(i)=tt;
mu(i) = log(s0) + (rd - rf - 0.5 * sigma ^ 2) * t(i);
sigmat(i) = sigma * sqrt(t(i));
end
for i = 1 : nav
    for j = 1 : nav
         covmat(i, j) = min(t(i), t(j))* sigma^ 2;
    end
end
ma = 0;
for i = 1 : nav
    ma = ma + exp((mu(i) + 0.5 * sigmat(i) ^ 2)) / nav; % Ariph
end
masq = 0;
for i = 1 : nav
    for j = 1 : nav
         masq = masq + exp(mu(i) + mu(j) + 0.5 * (sigmat(i) ^ 2 + sigmat(j) ^ 2 + 2
* covmat(i, j))) / nav ^ 2; %Ariph
    end
end
mmax = 2 * log(ma) - 0.5 * log(masq); % mean for log-normal approx of ariphmetic
sigmamax = sqrt(log(masq) - 2 * log(ma)); % sigma for log-normal approx of
ariphmetic
% ca-Call option price if ariphmet average approx log-norm
ca = exp(-rd * topt) * (exp(mmax + 0.5 * sigmamax ^ 2) * lp((sigmamax ^ 2 + mmax -
log(k)) / sigmamax) - k * lp((mmax - log(k)) / sigmamax));
pt = ca + (k - ma) * exp(-rd * topt);%Put
end
```

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```
function [N]=lp(x);
N=1/2*erfc(-x/sqrt(2));
end
end


function [cmc cmcmin cmcmax ] = AsianMC
%UNTITLED3 Summary of this function goes here
%   Detailed explanation goes here
s0=100;
k=100;
rd=0.09;
rf=0;
sigma=0.3;
t0='9 aug 2005';
tstart='10 aug 2005';
texp='10 aug 2006';
delt=7;
topt=(datenum(texp)-datenum(t0))/365;
nav=floor((datenum(texp)-datenum(tstart))/delt)+1;
for i=1:nav,
t(i)=(datenum(tstart)-datenum(t0)+(i-1)*delt)/365;
%t(i)=(td(i)-datenum(t0))/365;
mu(i)=log(s0)+(rd-rf-0.5*sigma^2)*t(i);
sigmat(i)=sigma*sqrt(t(i));
end
for i=1:nav
    for j=1:nav
        if t(i)~=0 || t(j)~=0
        ro(i,j)=sqrt(min(t(i),t(j))/max(t(i),t(j)));
    else
```

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```
        ro(i,j)=0;
    end
    end
end
c0=clock;
nmc=100000;
nn1=0;
nn2=0;
nn3=0;
nn4=0;
mm=0;
for i=1:nmc,
    x=normrndcor(mu',sigmat',ro);
    avx=mean(x);
    avS=mean(exp(x));
    if avx>log(k) && avS>=k
        nn1=nn1+1;
     elseif avx>log(k) && avS<k
        nn2=nn2+1;
     elseif avx<=log(k) && avS>=k
        nn3=nn3+1;
        price1(i)=avS-k;
     elseif avx<=log(k) && avS<k
       nn4=nn4+1;
    end
   if avS>=k
       price(i)=avS-k;
   else
       price(i)=0;
    end
```

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```
end
%nn1,nn2,nn3,nn4;
cmc=exp(-rd*topt)*mean(price)
cmcmin=cmc-3*sqrt((var(price))/nmc)*exp(-rd*topt)
cmcmax=cmc+3*sqrt((var(price))/nmc)*exp(-rd*topt)
c1=clock;
t1=(c1(6)-c0(6))/1
function [R]=normrndcor(mu,sigma,cormat)
% R = NORMRNDCOR(MU,SIGMA,CORMAT) returns a vector of random numbers chosen
%  from n-dimensional  normal distribution with parameters MU , SIGMA , CORMAT.
[n1,n2]=size(mu);
if n2~=1
    error('mean is not vector')
end
[n3,n4]=size(sigma);
if n4~=1
    error('sigma is not vector')
end
[n5,n6]=size(cormat);
if n1~=n3 || n5~=n1 || n6~=n1
    error('dimension is not correct')
end
covmat=diag(sigma)*cormat*diag(sigma);
%[U1,S]=eig(covmat);
%[U,Q]=qr(U1);
if sigma(1)~=0
%[U1,S]=eig(covmat);
%[U,Q]=qr(U1);
[U,S,V]=svd(covmat);
R=U*normrnd(zeros(n1,1),sqrt(diag(S)))+mu;
```

```
else
    for i=1:n1-1
        for j=1:n1-1
         covmat1(i,j)=covmat(i+1,j+1);
        end
 end
 [U,S,V]=svd(covmat1);
 R1=U*normrnd(zeros(n1-1,1),sqrt(diag(S)));
 R=[0;R1]+mu;
end
end
end
```